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CAUSALITY IN TOPOLOGICALLY NONTRIVIAL SPACE-TIMES[1]

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Abstract

The problems causality and causality violation in topologically nontrivial space-time models are considered. To this end the mixed boundary problem for traversable wormhole models is formulated and the influence of the boundary conditions on the causal properties of space-time is analyzed

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The problems of causality and causality violation in topologically nontrivial space-time models are considered in connection with (1) numerous declarations about "absurdly easy wormhole transformation into the time machine" [1, 2, 3, 4], which contradict to well-known theorems about global hyperbolicity and Cauchy problem [5, 6], and (2) statements about existence of physical "paradoxes" in the presence of time machine [7] as well as the statements about necessity of introduction for such models an additional "self-consistency conditions" to avoid these paradoxes [2].

To describe space-time model with traversable wormhole it is necessary to use minimum two maps: one (or more) map for description of the wormhole interior and one (or more) map for external space-time. Consider the simplest case when the space section of the wormhole has topology of the direct product $S^2 \times I$, where I is a closed interval $-L_1 < \xi^1 < L_2$ and S^2 is a two sphere with coordinates $(\xi^2,\ \xi^3)$. Let external space-time is described by coordinates $\{t,\ x^i\}$ and has metric

$$ds^2_{ext} = dt^2 - \gamma_{ij} dx^i dx^j, \tag{1}$$

and interior space-time, which is described by coordinates $\{\tau,\ \xi^j\}$, has metric

$$ds^2_{int} = a^2(\tau,\xi) d\tau^2 - 2b_i(\tau,\xi) d\tau d\xi^i - \widetilde{\gamma}_{ij}(\tau,\xi) d\xi^i d\xi_j, \tag{2}$$

where $-\infty < t,\ \tau < \infty$, and $a^2(\tau,\xi) > 0$ because of the supposition about traversability of the wormhole in both direction. In general case the correspondence between internal and external coordinates near the mouths of the wormhole are given by the following equations

$$t_{left} = \tau, \quad x^i_{left} = x^i_{left}(\xi^1,\xi^2,\xi^3), \qquad t_{right} = t_r(\tau), \quad x^i_{right} = x^i_{right}(\tau,\xi^1,\xi^2,\xi^3) \tag{3}$$

Equations (3) define the topology of space-time. They show in particular that for fixed τ wormhole connects the points of external space-like hypersurface $(t_1(\tau), M^3_{ext1})$ with the points of external space-like hypersurface $(t_2(\tau), M^3_{ext2})$, where $t_1 \neq t_2$ in general case. These equations induce the following boundary conditions for the components of internal metric of the wormhole:

$$a^2(\tau,\xi) = \begin{cases} 1 & \text{for } \xi^1 \to -L_1; \\ \alpha^2 \cdot (1 - v_i v^i) & \text{for } \xi^1 \to L_2, \end{cases} \tag{4}$$

$$\beta_i = \begin{cases} 0 & \text{for } \xi^1 \to -L_1 \\ \gamma_{kl} \frac{\partial x^k_{right}}{\partial \tau} \frac{\partial x^l_{right}}{\partial \xi^i} & \text{for } \xi^1 \to L_2, \end{cases} \tag{5}$$

and

$$\widetilde{\gamma}_{ij} = \begin{cases} \gamma_{kl} \frac{\partial x^k_{left}}{\partial \xi^i} \frac{\partial x^l_{left}}{\partial \xi^j} & \text{for } \xi^1 \to -L_1, \\ \gamma_{kl} \frac{\partial x^k_{right}}{\partial \xi^i} \frac{\partial x^l_{right}}{\partial \xi^j} & \text{for } \xi^1 \to L_2, \end{cases} \tag{6}$$

where $\alpha = dt_r/d\tau$, and $v_l = \frac{1}{\alpha} \gamma_{kl} \frac{\partial x^k_{right}}{\partial \tau}$.

Equations (3)-(6) together with condition $a^2(\tau,\xi) > 0$, which follows from the supposition about traversability of the wormhole in both direction, and standard initial conditions for internal and external metric, which near the left mouth must also satisfy to the conditions analogous to Eqs. (4)-(6), form the mixed boundary problem for Einstein equations. The mixed boundary problems for the source fields are formulated by analogous manner. In

general case these boundary problems cannot be reduced to Cauchy problem. So, the models with causality violation have non-evolutionary nature. In particular, in opposite to the statements [1, 2, 4] these models can not be considered as "transformation" of some initial configuration into the time machine.

It is easy to see that equations (4)- (6) and analogous conditions for other fields are the part of definition of the corresponding geometrical objects on manifold. In particular, they provide the self-consistency of solutions and the absence of any "paradoxes" in space-time models with causality violation.

On the other hand because of the non-evolutionary nature of space-time models with causality violation, which solve the general mixed boundary problem are formulated above, the question about the physical sense of models with causality violation is reduced to the question about the physical sense of boundary conditions (3)-(6), which must be given in general case in the points are separated bi time-like intervals. The existence of non-singular solutions of the above mixed boundary problem for space-time models with non-trivial causal structure is also unresolved problem.

It is clear that the causal properties of space-time with traversable wormhole are defined in general case both by the boundary conditions (3) and by the field equations. Nevertheless at least in two particular cases the causal structure of space-time is independent both on the motion of the mouths of the wormhole and on other physical processes.

Namely, it is easy to see that if $t_{left} = t_{right} = \tau$ in equations (3) then the causality violation is impossible independenty on the wormhole' mouths motion in the exterior space or on the other physical processes in space-time. Moreover, if the length of the wormhole's handle is short enough then in the external space-time (1) the absolute synchronization of events near the mouths of the wormhole as well as the preferable class of reference frames is exist; (2) the motion of the wormhole' mouths is absolute in difference with the motion of the material bodies. In particular, the same motion of the mouths as the bodies motion in well known "twins paradox" of special relativity does not lead to causality violation [8, 9] in opposite to conclusions of [1, 2].

Another particular case corresponds to the spherical wormhole with immovable mouths (i.e. if $x^i_{right} = x^i_{right}(\xi^1, \xi^2, \xi^3)$ in equations (3)) which is connected to external Minkowski space-time. Direct calculations show that in this case both the structure and properties of energy-momentum tensor in the wormhole's interior for the models with causality violations are the same as in non-causal case which were considered in [10]. Hence the causal structure of such model is defined completely by conditions (3).

Analysis of the other kinds of space-time models with causality violation shows that they also satisfy to the boundary or mixed boundary problem which is similar to the considered above and do not contain any paradoxes.

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